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International Journal of Heat and Mass Transfer 46 (2003) 1575–1586
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Effect of the history term on the transient energy equation for a sphere

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Received 18 June 2002; received in revised form 1 October 2002

Abstract

This paper addresses the importance of the history term on the transient energy equation of particles. The physical origin of this term is the diffusion of the transient temperature gradients around the sphere. The history term accounts for the effect of all the previous temperature changes of the sphere to the current temperature change. The derivation and analysis of the transient energy equation of rigid particles are presented. In order to solve numerically the transient energy equation, three different fluid temperature fields (step, ramp and sinusoidal) are applied and the temperature of the sphere is computed with and without the history term. The evaluation of the maximum deviation between these two computations allows us to determine for each case the effect of the history term and, especially, when the history term may be neglected. The final results of these computations allow several conclusions and recommendations on the appearance, importance and significance of the history term.

Keywords: Transient; History term; Particle; Drop

1. Introduction

The subject of heat and mass transfer from spheres has many practical engineering applications such as combustion, propulsion processes, chemical reaction, mixing and separation processes, boiling and condensation processes, and environmental sedimentation. At low Reynolds number flow (creeping flow), it is of importance in many extraction processes such as spray drying, aerosol scrubbing and aerosol/meteorological studies.

The classical problem of heat transfer from a single spherical particle moving in its surrounding fluid at low Reynolds number has been the subject of numerous investigations. Most of the theoretical work is connected with the case of steady-state heat transfer, in which the temperature of the particle is maintained constant and both the velocity and thermal fields around the particle are constant with respect to time. However, in many industrial applications the heat transfer between particles and the continuous phase is unsteady. The unsteadiness is due to the heating or cooling of the particles, bubbles and droplets and also to the time-dependent nature of the external flow [3].

The first study on the transient rate of heat transfer from a sphere was performed by Fourier [6] who used his results to determine the age of the earth. In doing so, he defined conductivity and set the foundations of the field of heat and mass transfer. Carslaw and Jaeger [2] extended Fourier's ideas on the transient conduction from a solid sphere and presented several different applications of transient heat transfer at very low Peclet numbers. More recently, Cooper [4] and Brunn [1] examined two problems related to the heat conduction from a sphere to an infinite medium, when the initial temperatures of the sphere and the medium are different. Most of the results on this subject were restricted to specific thermal processes with practical applications, such as step temperature change or sinusoidal temperature variation [12]. A general expression for an arbitrary temperature field in the surrounding fluid or in the interior of the sphere was not studied until recently by Michaelides and Feng [10]. They obtained the first

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PII: S0017-9310(02)00426-X

Nomenclature

Latin symbols

a	heat diffusivity
c	specific heat capacity
erf	error function
erfc	complementary error function
k	conductivity
m	mass
Nu	Nusselt number
Pe	Peclet number
Q	rate of heat transfer
r	coordinate relative to particle
R	radius of the fluid domain
Sl	Strouhal number
t	time
T	temperature
u	fluid velocity
x	Eulerian coordinate system
z	coordinates moving with the sphere

Greek symbols

α	radius of the sphere
β	ratio of the fluid to solid volumetric heat capacities
ρ	density
τ	dummy variable with units of time
τ_f	characteristic time of fluid
θ	phase difference
σ	dummy variable dimensionless time
ε	phase angle (energy)
ω	dimensional frequency

Superscripts

$*$	dimensionless
0	pertains to undisturbed field

Subscripts

f	pertains to fluid
i	vector component
jj	Laplacian
s	pertains to sphere
with	with history term
without	without history term

complete analytical solution for the unsteady energy equation at creeping flow conditions that correspond to the transient conduction. They showed for the first time that the transient energy equation, at creeping flow also contains a history term, which is expressed mathematically by an integral. Physically, the unsteady heat transfer from a sphere is influenced by the past thermal history of the sphere as well as the temperature gradients of the surrounding fluid [11,12].

The objective of this paper is to determine the effect of the history term on the transient energy equation of a sphere. Since the solution of the transient energy equation is rather complicated with the presence of the history term, it is useful to know if it is advisable or not to neglect this term before solving this problem of heat or mass transfer; and furthermore, if this term is neglected, what is the error one should expect in the computations. If the history term can be neglected, the transient energy equation, which is a first-order integro-differential equation, becomes an ordinary first-order differential equation and its solution may be obtained relatively easily. On the other hand, if the history term is important and must be retained, the computations are more cumbersome and time consuming.

We have considered the transient heat transfer cases related to three different fluid temperature processes: step temperature change, ramp temperature change and sinusoidal temperature change. The influence of the history term has been determined by computing the transient energy equation with and without the history term. The calculations allow us to make conclusions and recommendations on the importance of the history term and, especially, under what conditions it must be retained or may be neglected.

2. The transient energy equation for a sphere

2.1. The case of creeping flow, $Pe \ll 1$

The exact solution of the governing equation of heat transfer can be obtained only for creeping flow conditions ($Pe \ll 1$), that is, when the heat transfer is characterized only by the transient conduction equation. Michaelides and Feng [10] conducted a study for the transient heat transfer from a solid sphere with uniform temperature $T_s(t)$. They derived the following expression for the Lagrangian energy equation of a sphere, which is present in an arbitrary fluid temperature field $T_f(z,t)$ with the initial condition $T_s(0) = T_f^0(0)\big|_{z=0}$:

$$m_s c_s \frac{dT_s(t)}{dt} = m_f c_f \frac{DT_f^0(z,t)}{Dt}\bigg|_{z=0} - 4\pi\alpha k_f \left[T_s(t) - T_f^0(z,t)\big|_{z=0} - \frac{1}{6}\alpha^2 T^0_{f,jj}(z,t)\bigg|_{z=0} \right] - 4\pi\alpha^2 k_f \times \int_0^t \frac{\frac{d}{d\tau}\left[T_s(t) - T_f^0(z,t)\big|_{z=0} - \frac{1}{6}\alpha^2 T^0_{f,jj}(z,t)\big|_{z=0} \right]}{\sqrt{\pi a_f (t-\tau)}} d\tau \quad (1)$$

where m_s is the mass of the sphere, m_f the mass of the fluid in an equal volume as the one of the particle, c_s the specific heat capacity of the sphere and c_f the one of the fluid, is the radius of the sphere, k_f is the conductivity of the fluid, $T_s(t)$ is the temperature of the sphere, $T_f^0(z,t)|_{z=0}$ is the undisturbed temperature field of the fluid evaluated at the center of the sphere, a_f is heat diffusivity of the fluid, t is the time and τ is a dummy variable with units of time. It must be emphasized that Eq. (1) is derived by the complete analytical solution of the conduction equation in the presence of a sphere. While the domain and mass of the sphere is finite and restricted to the space $0 < r < \alpha$, the mass of the surrounding fluid is by far greater, because the characteristic dimension of the fluid L is assumed much greater than the radius of the sphere $\alpha \ll L$. Details on the derivation of Eq. (1) may be found in [9,10].

From the above equation, the history term associated with the process of heat transfer at creeping flow conditions is as follows:

$$4\pi\alpha^2 k_f \int_0^t \frac{\frac{d}{d\tau}\left[T_s(t) - T_f^0(z,t)\big|_{z=0} - \frac{1}{6}\alpha^2 T^0_{f_{,jj}}(z,t)\Big|_{z=0}\right]}{\sqrt{\pi a_f(t-\tau)}} d\tau \tag{2}$$

It must be mentioned that although the study by Michaelides and Feng [10] was the first to point out the existence of the history term, an allusion to this term for a few very simple transient conduction problems has appeared in some other studies including the book of Carslaw and Jaeger [2]. They found that the solution for the transient conduction with a step temperature change is expressed in terms of the error function, which is essentially an integral of the history of the process.

The energy equation is made dimensionless by using the thermal timescale of the fluid $\tau_f = \alpha^2\rho_f c_f/k_f$. Also, a dimensionless parameter is introduced, $\beta = \rho_f c_f/\rho_s c_s$, which is the ratio of the volumetric heat capacities. Since the sphere is considered small compared to the fluid, the Laplacian can be neglected. Hence, the dimensionless energy equation becomes:

$$\frac{dT_s^*}{dt^*} = \beta \frac{dT_f^{0^*}}{dt^*}\bigg|_{z=0} - 3\beta\left(T_s^* - T_f^{0^*}\big|_{z=0}\right) - \frac{3\beta}{\sqrt{\pi}}\int_0^{t^*} \frac{\frac{d}{d\sigma}\left[T_s^* - T_f^{0^*}\big|_{z=0}\right]}{\sqrt{t^*-\sigma}} d\sigma \tag{3a}$$

where σ is a dummy variable defined as $\sigma = \tau/\tau_f$ and the dimensionless initial condition for the temperature is:

$$T_s^*(0) = T_f^{0^*}(0)\big|_{z=0} \tag{3b}$$

Eq. (3a) is an integro-differential equation and is implicit for the variable T_s. Its solution must involve an implicit numerical method, which is always time consuming and requires a great deal of CPU and memory. This equation may be transformed to yield a second-order ordinary differential equation, which is explicit in T_s. The transformation eliminates the need for iterations in and reduces considerably the memory requirements.

The derivation of the second-order ordinary differential equation has been done by following a method by Michaelides [8] for the history term applied to the equation of motion. The derivation is accomplished by taking the Laplace transform of the first-order integro-differential equation (1), doing algebraic manipulations in the Laplace domain and transforming back to the time domain. The final expression thus derived is as follows:

$$\begin{aligned}
&\frac{\alpha^4\rho_s^2c_s^2}{k_f^2}\frac{d^2\left(T_s - T_f^0\big|_{z=0}\right)}{dt^2} + \frac{\alpha^2\rho_s c_s}{k_f}(6-9\beta)\frac{d\left(T_s - T_f^0\big|_{z=0}\right)}{dt} + 9(T_s - T_f^0\big|_{z=0}) \\
&= -\frac{\alpha^4\rho_s^2c_s^2}{k_f^2}(1-\beta)\frac{d^2T_f^0}{dt^2}\bigg|_{z=0} - \frac{3\alpha^2\rho_s c_s}{k_f}(1-\beta)\frac{dT_f^0}{dt}\bigg|_{z=0} \\
&+ 3(1-\beta)\frac{\alpha^2\rho_s c_s}{k_f}\sqrt{\frac{\alpha^2\rho_f c_f}{k_f}}\int_0^t \frac{\frac{d^2T_f^0}{d\tau^2}\Big|_{z=0}}{\sqrt{\pi(t-\tau)}}d\tau \\
&+ 3(1-\beta)\frac{\alpha^2\rho_s c_s}{k_f}\sqrt{\frac{\alpha^2\rho_f c_f}{\pi t k_f}}\frac{dT_f^0}{dt}\bigg|_{\substack{z=0\\t=0}}
\end{aligned} \tag{4a}$$

with initial conditions:

$$T_s(0) = T_f^0(0)\big|_{z=0} \tag{4b}$$

and

$$\frac{dT_s(0)}{dt} = \beta\frac{dT_f^0(0)}{dt}\bigg|_{z=0} \tag{4c}$$

The equation may be made dimensionless, by using the characteristic time of the fluid τ_f and the parameter β:

$$\begin{aligned}
&\frac{1}{\beta^2}\frac{d^2\left(T_s^* - T_f^{0^*}\big|_{z=0}\right)}{dt^{*2}} + \frac{(6-9\beta)}{\beta}\frac{d\left(T_s^* - T_f^{0^*}\big|_{z=0}\right)}{dt^*} + 9\left(T_s^* - T_f^{0^*}\big|_{z=0}\right) \\
&= -\frac{(1-\beta)}{\beta^2}\frac{d^2T_f^{0^*}}{dt^{*2}}\bigg|_{z=0} - \frac{3(1-\beta)}{\beta}\frac{dT_f^{0^*}}{dt^*}\bigg|_{z=0} \\
&+ \frac{3(1-\beta)}{\beta}\int_0^{t^*}\frac{\frac{d^2T_f^{0^*}}{d\sigma^2}\Big|_{z=0}}{\sqrt{\pi(t^*-\sigma)}}d\sigma \\
&+ \frac{3(1-\beta)}{\beta}\sqrt{\frac{1}{\pi t^*}}\frac{dT_f^{0^*}}{dt^*}\bigg|_{\substack{z=0\\t^*=0}}
\end{aligned} \tag{5a}$$

with the dimensionless initial conditions:

$$T_s^*(0) = T_f^{0^*}(0)\big|_{z=0} \tag{5b}$$

$$\frac{dT_s^*(0)}{dt^*} = \beta\frac{dT_f^{0^*}(0)}{dt^*}\bigg|_{z=0} \tag{5c}$$

The only disadvantage of this equation is that it is a second-order differential equation and that it contains more terms than Eq. (3a). Since it is a second-order equation, the question of numerical stability comes into consideration. We have found out that the last equation becomes numerically unstable for $\beta > 2/3$. In this case one has to solve the original integro-differential equation (1) or (3).

2.2. The case of small but finite Peclet number, $Pe < 1$

Feng and Michaelides [5] conducted an asymptotic study on the heat transfer from a sphere at finite but small Peclet numbers ($Pe < 1$) in the case of a step temperature change. They have found out that at short times from the inception of the process all energy exchange is confined to the vicinity of the particle and conduction is the dominant mode of heat transfer. In this time domain, they obtained the following expression for the total rate of heat transfer and the Nusselt number:

$$Q(t) = -4\pi(T_s - T_f^0) - 4\pi(T_s - T_f^0)\frac{1}{\sqrt{t}} + o(Pe^{1+}) \qquad (6a)$$

and

$$Nu(t) = -\frac{Q(t)}{2\pi(T_s - T_f^0)} = 2\left[1 + \frac{1}{\sqrt{t}}\right] + o(Pe^{1+}) \qquad (6b)$$

The rate of approach to steady state is proportional to $t^{-1/2}$, as expected in this purely diffusional process and does not depend on Pe, which is a measure of the inertia/advection of the sphere.

At long times after the commencement of the process, heat transfer by advection becomes important, while close to the surface of the sphere, conduction dominates. In this case, close-form solutions for the heat transfer may be obtained for specific processes: In the case of a step temperature change for the fluid, and for the long-time domain, the total rate of heat transfer and the corresponding Nusselt number become:

$$Q(t) = -4\pi(T_s - T_f^0) - 4\pi(T_s - T_f^0)\left[\frac{\exp\left(-\frac{Pe^2}{4}t\right)}{\sqrt{\pi t}} + \sqrt{\frac{Pe^2}{4}}\operatorname{erf}\sqrt{\frac{Pe^2}{4}t}\right] + o(Pe^{1+}) \qquad (7a)$$

and

$$Nu(t) = 2 + 2\left[\frac{\exp\left(-\frac{Pe^2}{4}t\right)}{\sqrt{\pi t}} + \sqrt{\frac{Pe^2}{4}}\operatorname{erf}\left(\sqrt{\frac{Pe^2}{4}t}\right)\right] + o(Pe^{1+}) \qquad (7b)$$

It is obvious that the rate of approach to the steady-state solution is $e^{-t}t^{-1/2}$. The approach to steady state during convection is faster than the approach during the purely conduction mode, which is $t^{-1/2}$, because the thermal wake is well formed in the outer region and is spreading by advection. The spread of the thermal wake facilitates the exchange of energy in the outer region of the fluid and, hence, accelerates the approach to steady state. It must be pointed out that similar results for the equation of motion of the sphere at creeping flow conditions and at small but finite Reynolds numbers were obtained by Lovalenti and Brady [7].

Numerical calculations were performed to determine the Nusselt number as functions of Pe in simple cases. Fig. 1a shows the transient Nusselt number in the short time domain in the case of sphere in an otherwise quiescent fluid for $Pe = 0.25$. The temperature of the sphere undergoes a step temperature change at time $t = 0$. It is observed that Nu initially declines rapidly, and then approaches the steady-state solution ($Nu = 2$) at a very slow rate. The solution at long-time domain is shown in Fig. 1b, where it is apparent that the Nusselt number approaches asymptotically the steady-state value $Nu = 2.25$. The last figure is based on expression (7b).

3. Results and discussion

In order to deduce the effect of the history term and its importance on the calculations we have made several calculations on the influence of the history term under different scenarios of fluid temperature variation. We placed emphasis on the computations with the creeping flow form of the energy equation for two reasons: (a) the history term decays as $t^{-1/2}$ and, thus, persists longer, and (b) in the case of finite Pe, the influence of the history/transient term is easier to obtain by evaluating the various terms of the derived energy equation, such as Eqs. (7a) and (7b). The computations on the temperature of the sphere were performed for the following cases of fluid temperature variation:

- a step temperature change,
- a ramp increase of the fluid temperature,
- a sinusoidal temperature variation.

Whenever it was possible we solved the second-order o.d.e. (Eq. (4a)) to obtain the variation of the temperature of the sphere. Otherwise, (when Eq. (4a) was unstable) we solved the integro-differential equation (1). In this case, the evaluation of the history term was accomplished by using the following expansion [13]:

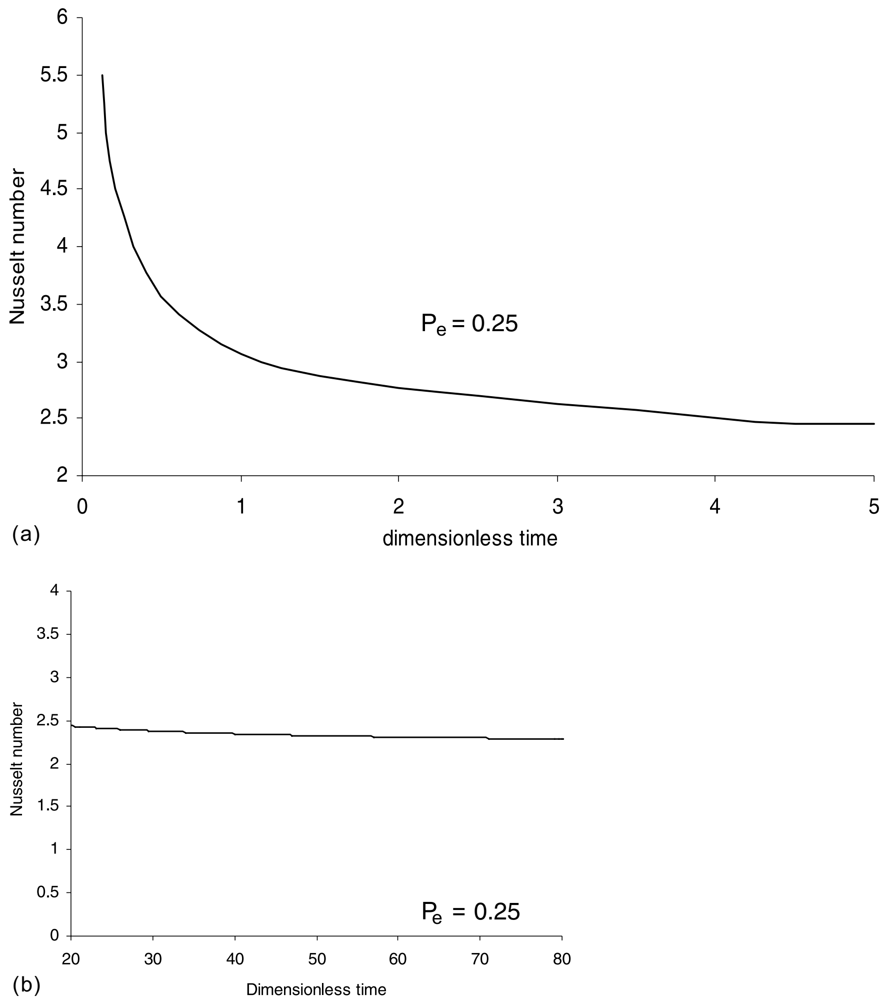

Fig. 1. (a) Nusselt number versus dimensionless time for the conduction solution at short time with a step temperature change for $Pe = 0.25$. (b) Nusselt number versus dimensionless time for the conduction solution in the long-time domain with a step temperature change for $Pe = 0.25$.

$$\int_0^{nh} \frac{\frac{dT_f^0}{d\sigma}}{\sqrt{t-\sigma}}\,d\sigma = 2\sqrt{h}\sum_{i=1}^{n}\left(\overline{\frac{dT_f^0}{dt}}\right)_i \times\left[(n-i+1)^{1/2}-(n-i)^{1/2}\right] \tag{8a}$$

where the average temperature change is given as:

$$\left(\overline{\frac{dT_f^0}{dt}}\right)_i = \frac{1}{2}\left[\left(\frac{dT_f^0}{dt}\right)_i + \left(\frac{dT_f^0}{dt}\right)_{i-1}\right] \tag{8b}$$

In all the computations, a fourth-order Runge–Kutta technique was used. A variable time step was used in the computations: Initially ($t = 0$) a dimensionless time step equal to 0.0001 was taken and this was gradually increased to 0.01. There were no computational difficulties associated with the calculation of the integral term, other than the additional CPU time and memory usage. Specific details of the computations pertaining to the three cases we examined are given in the following sections.

3.1. Step temperature change

In a spherical system of coordinates, the energy equation of the fluid is:

$$\frac{\partial T_f}{\partial t} = a_f\left(\frac{\partial^2 T_f}{\partial r^2} + \frac{2}{r}\frac{\partial T_f}{\partial r}\right) \tag{9}$$

We assume that the center of the solid sphere is located at $r = 0$ and its radius is α. Also that the temperature of the fluid undergoes a step temperature change of (dimensionless) unit magnitude at the far field, which is at $R = 10\alpha$. From [2] we obtained the following expression for the dimensionless temperature field of the fluid that undergoes a step temperature change, in the domain $0 < r < 10\alpha$:

$$T_f(t,r) = \frac{R}{r}\sum_{i=0}^{\infty}\left(\operatorname{erfc}\left(\frac{(2i+1)R-r}{2\sqrt{a_f t}}\right) - \operatorname{erfc}\left(\frac{(2i+1)R+r}{2\sqrt{a_f t}}\right)\right) \quad (10a)$$

By taking the limit at the center of the sphere, we obtain the fluid temperature at $r = 0$ in the absence of the sphere (which is needed in the computations) as follows:

$$T_f^*(t^*)\big|_{r=0} = \frac{2R}{\alpha\sqrt{\pi t^*}}\sum_{i=0}^{\infty}\left(\exp\left(-\frac{(2i+1)^2R^2}{4\alpha^2 t^*}\right)\right) \quad (10b)$$

Using the last two equations we computed the resulting temperature of the sphere with and without the history term. Fig. 2, depicts the temperature of the fluid at the far field and at $r = 0$. It also depicts the temperature of the sphere by using and by not using the history term for $\beta = 0.01$. It is observed that the addition of the history term in the computations enhances the heat transfer to the sphere and shortens the time to reach equilibrium. In order to quantify the effect of the history term we define the percentage deviation at time t between the response to the fluid with the history term and the one without the history term:

$$\text{deviation } (\%) = \frac{|T_{s_{\text{with}}} - T_{s_{\text{without}}}|}{|T_f|} \times 100 \quad (11)$$

The maximum percentage deviation obtained in the computations has been recorded and plotted in Fig. 3 as a function of the parameter β. It is observed that the maximum deviation is less than 10% in all the cases and that it only exceeds 2.5% in the range $0.0015 < \beta < 0.2$, which corresponds to the case of heat transfer from drops and particles in gases as well as of particles in light liquids. In these cases, neglecting the history term in the computations will result in significant errors (greater than 2.5%) on the temperature of the particle or droplet and the total heat or mass exchange.

3.2. Ramp temperature change

In this case, the dimensionless temperature of the fluid is driven by a ramp temperature change at the far field, equal to kt. The temperature of the fluid in the domain $0 < r < 10\alpha$ is given in a dimensionless form by the following expression [2]:

$$T_f(t,r) = k\left[t - \frac{R^2 - r^2}{6a_f}\right] - \frac{2kR^3}{a_f\pi^3 r}\sum_{i=1}^{\infty}\frac{(-1)^i}{i^3} \times \exp\left(-\frac{a_f i^2\pi^2 t}{R^2}\right)\sin\left(\frac{i\pi r}{R}\right) \quad (12a)$$

When the time becomes dimensionless by using the thermal timescale of the fluid and a Taylor expansion is made close to the origin, we obtain the following expression for the dimensionless fluid temperature at $r = 0$ in the absence of the sphere:

$$T_f^*(t^*)\big|_{r=0} = k\tau_f\left[t^* - \frac{R^2}{6\alpha^2}\right] - \frac{2k\tau_f R^2}{\pi^2\alpha^2}\sum_{i=1}^{\infty}\frac{(-1)^i}{i^2} \times \exp\left(-\frac{i^2\pi^2\alpha^2 t^*}{R^2}\right) \quad (12b)$$

We performed similar computations in this case as in the case of the step temperature increase. The results are plotted in Fig. 4 for $\beta = 0.01$. As in the previous case, we observe that the inclusion of the history term actually

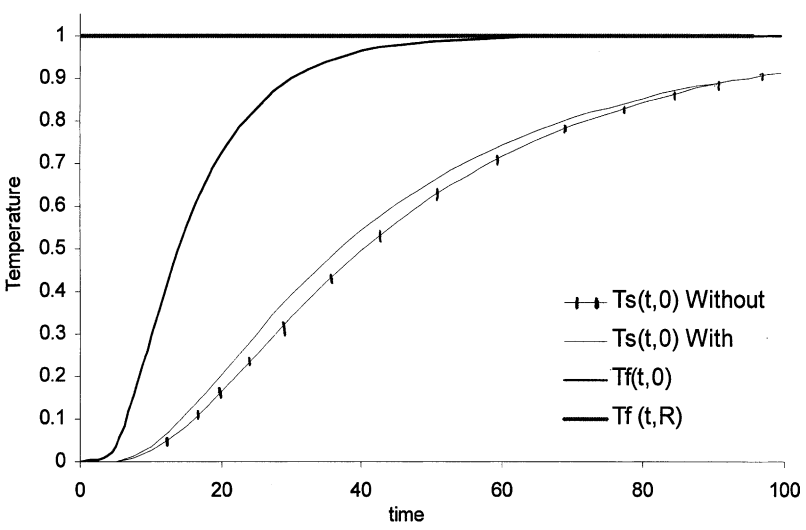

Fig. 2. Response to the step temperature change of the fluid for $\beta = 0.01$.

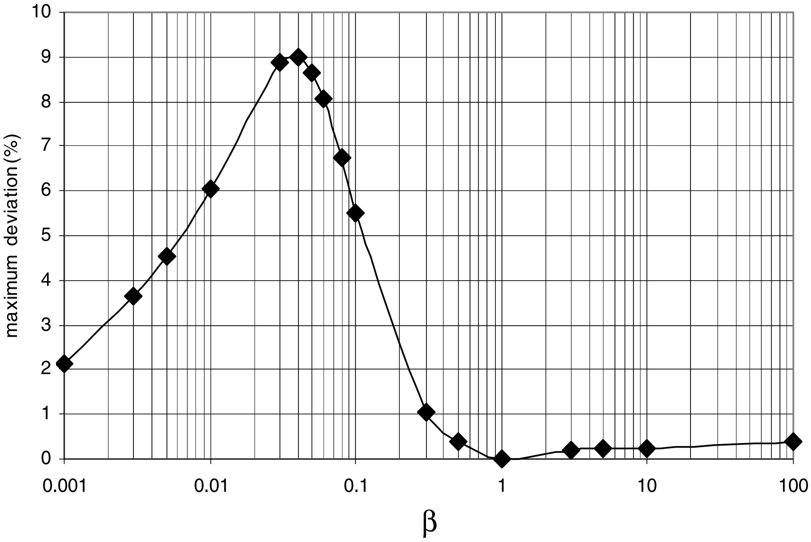

Fig. 3. Maximum deviation for the step temperature change versus β.

enhances the heat transfer to the sphere and that the temperature of the sphere follows faster the temperature of the fluid. The maximum deviation observed is plotted in Fig. 5, where it is noted that the overall error by neglecting the history term does not exceed 8%. The error is significant (more than 2.5%) in the range $0.0018 < \beta < 0.18$, which again corresponds to particle/droplet flows in gases or light liquids.

3.3. Sinusoidal temperature change

The response of the sphere surrounded by a fluid with a sinusoidal temperature variation in the far field is solved with zero initial temperature. The dimensionless far field temperature (at $R = 10\alpha$) varies as $\sin(\omega t + \varepsilon)$ where ω is the frequency of the variation and ε the phase. The dimensionless temperature in the case of a sinusoidal temperature change is given by Carslaw and Jaeger [2] as follows:

$$T_f(t,r) = \frac{RA}{r}\sin(\omega t + \varepsilon + \phi) + \frac{2Ra_f\pi}{r} \times \sum_{i=1}^{\infty} \frac{(-1)^i i(a_f i^2\pi^2 \sin\varepsilon - \omega R^2 \cos\varepsilon)}{a_f^2 i^4\pi^4 + \omega^2 R^4} \times \exp\left(\frac{-a_f i^2\pi^2 t}{R^2}\right)\sin\left(\frac{i\pi r}{R}\right) \quad (13a)$$

where

$$A = \left\{\frac{\cosh(2\omega' r) - \cos(2\omega' r)}{\cosh(2\omega' R) - \cos(2\omega' R)}\right\}^{1/2},$$

$$\phi = \arg\left\{\frac{\sinh(\omega' r(1+i))}{\sinh(\omega' R(1+i))}\right\} \quad \text{and} \quad \omega' = \sqrt{\frac{\omega}{2a_f}} \quad (13b)$$

When a dimensionless time is used, the following expression is obtained for the variation of the fluid temperature at the origin, in the absence of the sphere:

$$T_f^*(t^*)\big|_{r=0} = \frac{2\omega' R}{(\cosh(2\omega' R) - \cos(2\omega' R))^{1/2}} \times \sin(Sl\cdot t^* + \varepsilon + \phi) + \sum_{i=1}^{\infty}\frac{2(-1)^i(\alpha i\pi)^2((\alpha i\pi)^2\sin\varepsilon - Sl\cdot R^2\cos\varepsilon)}{\alpha^4 i^4\pi^4 + Sl^2R^4} \times \exp\left(\frac{-(\alpha i\pi)^2 t^*}{R^2}\right) \quad (13c)$$

where

$$\phi = tg^{-1}\left\{\frac{\sinh(\omega' R)\cos(\omega' R) - \cosh(\omega' R)\sin(\omega' R)}{\sinh(\omega' R)\cos(\omega' R) + \cosh(\omega' R)\sin(\omega' R)}\right\},$$

$$\omega' = \frac{1}{\alpha}\sqrt{\frac{Sl}{2}} \quad \text{and} \quad Sl = \omega\cdot\tau_f \quad (13d)$$

Because the fluid field has its own timescale of variation (ω^{-1}) a Strouhal number, Sl, appears in this case, which is the ratio of the thermal timescale of the fluid to the timescale of the variation of its temperature at the far field. The Strouhal number is the inverse of the Stokes number, which is normally used in the equation of motion computations.

As in the previous two cases, we computed the temperature of the sphere with and without the history term. Fig. 6 shows the dimensionless fluid and sphere temperatures (with and without the history term) for $\beta = 0.01$ and $Sl = 0.1$. It is obvious that in both cases the particle follows a sinusoidal temperature variation

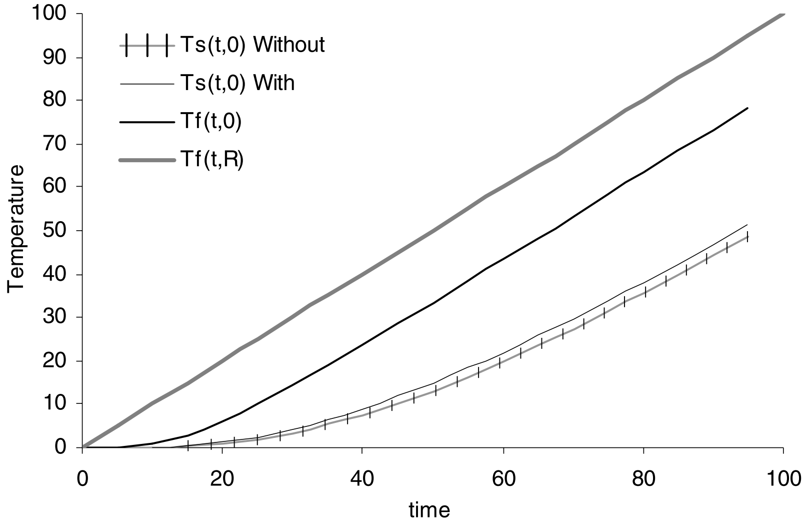

Fig. 4. Response to the ramp temperature change of the fluid for $\beta = 0.01$.

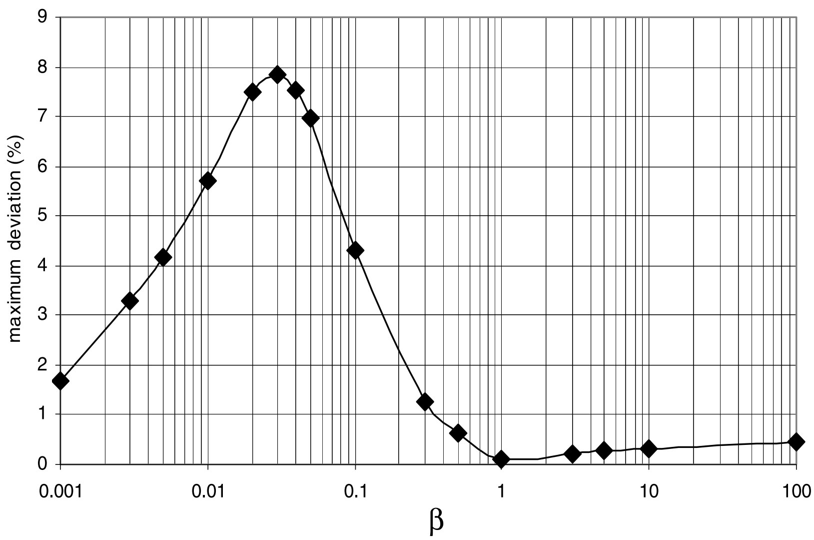

Fig. 5. Maximum deviation for the ramp temperature change versus β.

and that it lags the temperature of the fluid and the amplitude of its temperature variation is lower. It is also apparent that the inclusion of the history term in the transient energy equation has an effect on both the amplitude of the temperature and the phase lag of the particle with respect to the fluid. For this reason, both the maximum deviation for the amplitude and the phase ratio are computed. The results are shown in Figs. 7–10, where the maximum deviation in the calculations was plotted on a function of β for two value of the Strouhal number, $Sl = 0.001$ and $Sl = 0.1$. It is observed that when the rate of variation of the fluid temperature is very slow ($Sl = 0.001$) neglecting the history term does not influence significantly the computations. The variability of the fluid temperature field is slow enough for the transient term to be insignificant in the computations for the temperature of the sphere. In this case the process may be considered quasi-steady. This is corroborated by the results depicted on Fig. 8, where it is shown that the resulting phase change difference is also insignificant.

However, when the Strouhal number is higher the process cannot be considered quasi-steady, and the inclusion of the history (or any other transient) term plays

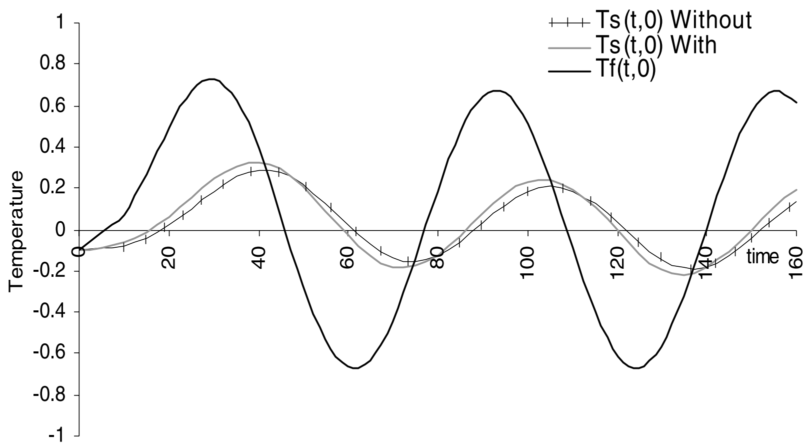

Fig. 6. Response to the sinusoidal temperature change of the fluid for $\beta = 0.01$ and $Sl = 0.1$.

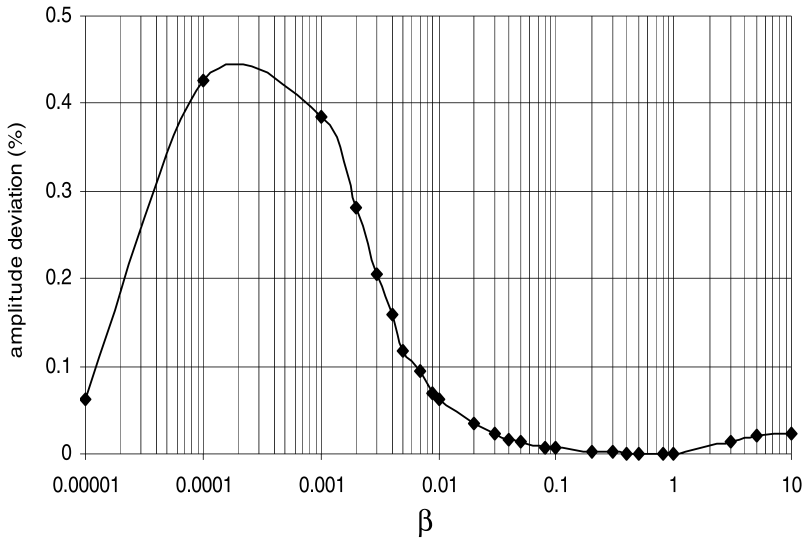

Fig. 7. Maximum amplitude deviation for the sinusoidal temperature change ($Sl = 0.001$) versus β.

an important role. Figs. 9 and 10 show the maximum amplitude deviation and the phase ratio for a sinusoidal fluid temperature variation, when the Strouhal number is equal to 0.1. It is obvious that the inclusion of the history term plays an important role in the computations, especially in the case of the phase ratio, which is defined in terms of the phase angle ϕ as follows:

$$\text{phase ratio} = \frac{\phi_{\text{f}} - \phi_{\text{with}}}{\phi_{\text{f}} - \phi_{\text{without}}} \tag{14}$$

In the case of the sinusoidal variation of the fluid temperature, it appears that the inclusion of the history term in the computations is important at high values of the Strouhal number and that the range of β that corresponds to particle/droplet flows in gases is more sensitive for the inclusion of this transient term.

From the physical point of view, as the temperature of the fluid at the far field follows a sinusoidal variation, the temperature of the sphere also follows a sinusoidal variation. Consequently the sphere will heat when the fluid temperature goes up and cool when the sinusoid goes down. The phase difference signifies that the heating or cooling process will begin with a time difference with respect to the fluid. The difference in the amplitude creates a reduction of the amount of heat received or rejected in the response of the sphere. For example, for $\beta = 0.01$ and $Sl = 0.1$ (Fig. 6) the dimensionless amount of heat given by the fluid is 0.8, then the amount of heat

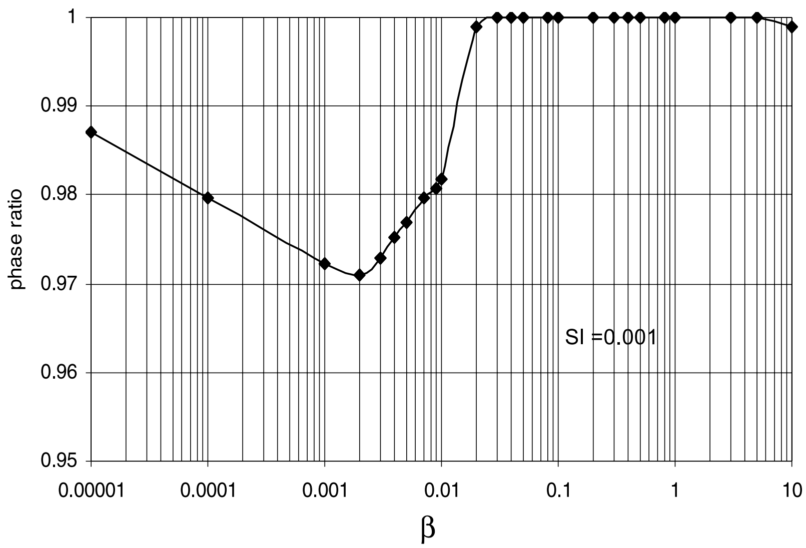

Fig. 8. Phase ratio for the sinusoidal temperature change ($Sl = 0.001$) versus β.

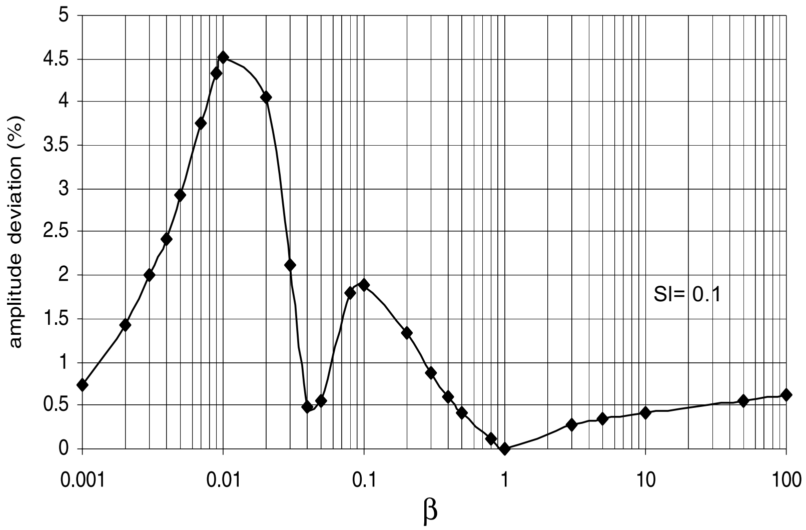

Fig. 9. Maximum amplitude deviation for the sinusoidal temperature change ($Sl = 0.1$) versus β.

received by the sphere is only 0.3 (with the history term) or less (without the history term). It means that less heat is transferred from the fluid to the sphere. When the history term is taken into account, the phase difference with the fluid is reduced and the amount of heat transferred is more, signifying an enhancement in the heat transfer.

It appears that as the Strouhal number increases the importance of the history term would increase too. However, high values of Sl signify that the temperature of the fluid in the far field heats and cools faster. The rate of heating and cooling may become too fast for the changes to be felt in the interior region, where the sphere is. Actually, if the variation at $r = 10\alpha$ is too fast, the change at $r = 0$ would be imperceptible and the sphere would not be affected. We have found out that if the Strouhal number is high ($Sl > 0.2$), the temperature of the sphere does not follow a sinusoidal function and the computations become meaningless. This is shown in Fig. 11, which has been computed with $Sl = 1$ and where the temperatures are shown after the passage of a significant amount of time ($t > 240$). In this case, the process is too fast for the temperature of the sphere to be dynamically stationary and, therefore, the calculations of the ampli-

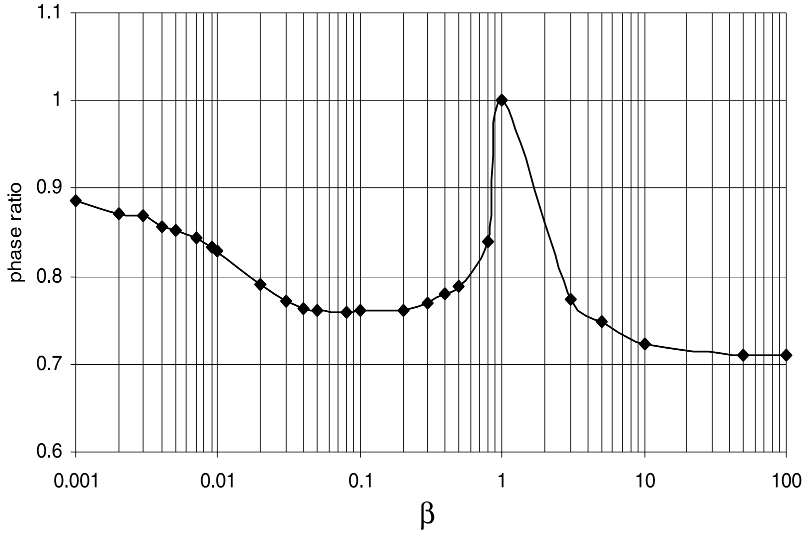

Fig. 10. Phase ratio for the sinusoidal temperature change ($Sl = 0.1$) versus β.

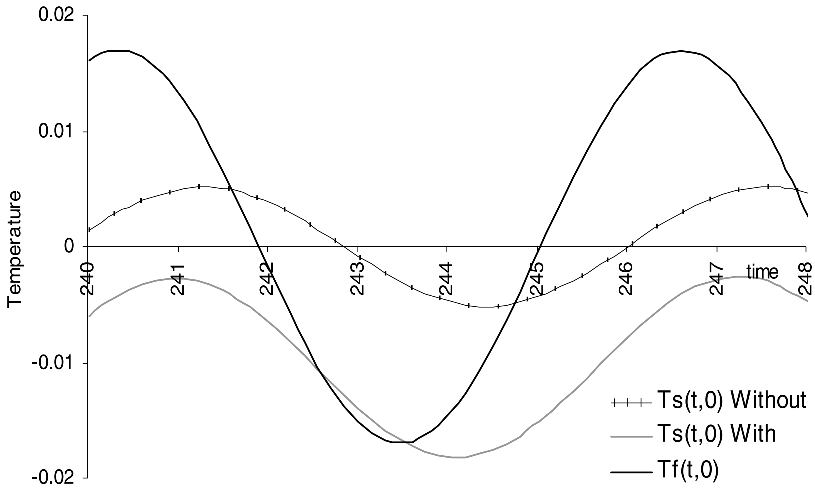

Fig. 11. Response to the sinusoidal temperature change of the fluid for $\beta = 0.1$ and $Sl = 1.0$.

tude ratio and phase difference are not physically meaningful. A glance at the figure though convinces that the inclusion of the history term plays a very important role in the computations.

4. Conclusions

The exact derivation of the transient energy equation of a sphere at creeping flow conditions reveals the presence of a history term, which is analogous to the history term of the equation of motion and accounts for the effect of all the previous temperature changes of the sphere. Thus, the transient energy equation becomes a first-order integro-differential equation. By using a transformation, this equation may be converted to a second-order ordinary differential equation, which is explicit in the temperature of the sphere.

Calculations were performed for three different fluid temperatures changes: a step temperature change, a ramp temperature change and a sinusoidal temperature change. The analysis of the results showed that the history term plays an important role in the computations involving mixtures of solids or drops in gases or light liquids, where the thermal capacity ratio β is between 0.002 and 0.2. In the case of sinusoidal temperature variation, the history term is also very important in the case of high Strouhal numbers. Results at the high values of the volumetric heat capacity ratio, β, show that the effect of the history term is not significant. Therefore, the case of bubbly flows in liquids, one may neglect the history term in Lagrangian computations.

Acknowledgements

The original work on this subject was supported by grants from the National Science Foundation and the Louisiana Education Quality Support Fund to Tulane University for which the authors are thankful. This work was also partly supported by the Tulane-Xavier Center for Bioenvironmental Research.

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